Chapter 10
Circles

Section 4
Other Angle Relationships in Circles

GOAL 1: Using Tangents and Chords

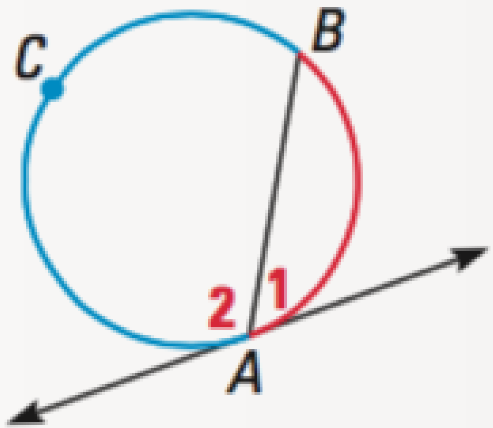
You know that the measure of an angle inscribed in a circle is half the measure of its intercepted arc. This is true even if one side of the angle is tangent to the circle. You will be asked to prove Theorem 10.12 in Exercises 37–39.

THEOREM

THEOREM 10.12

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

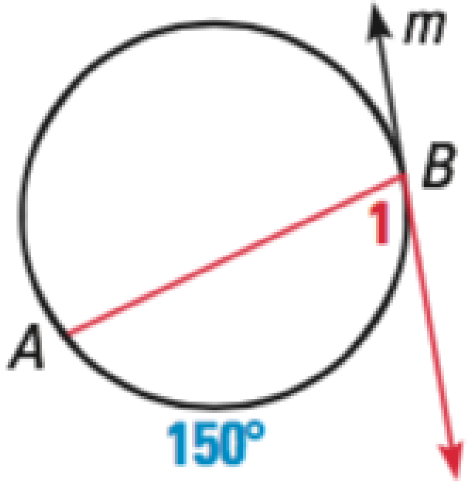
$m\angle 1 = \frac{1}{2} m\overset{\frown}{AB}$ $\qquad$ $m\angle 2 = \frac{1}{2} m\overset{\frown}{BCA}$



Example 1: Finding Angle and Arc Measures

Line m is tangent to the circle. Find the measure of the red angle or arc.

a.

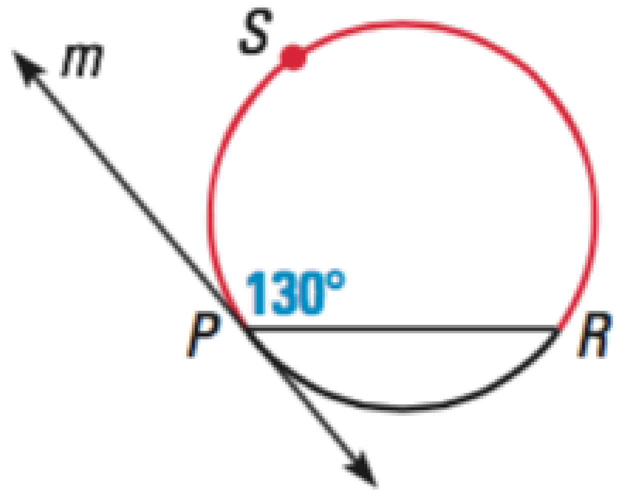


$\angle = \frac{1}{2} \frown$

$\angle = \frac{1}{2}(150)$

$\angle = 75^\circ$

b.



$\angle = \frac{1}{2} \frown$

$130 = \frac{1}{2} \frown$

$260 = \frown$

Example 2: Finding an Angle Measure

In the diagram below, BC is tangent to the circle. Find m<CBD.

$\angle = \frac{1}{2} \frown$

$5x = \frac{1}{2}(9x + 20)$

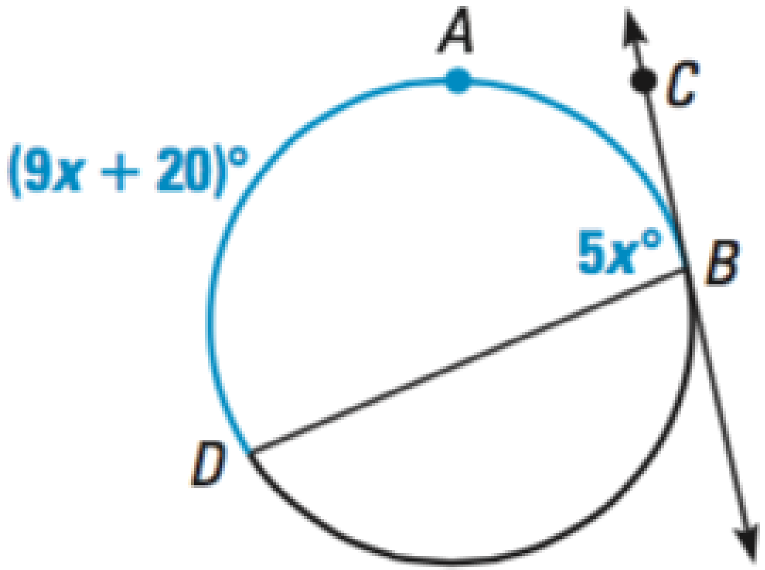
$5x = 4.5x + 10$

$-4.5x \quad \cancel{-4.5x}$

$\frac{.5x}{.5} = \frac{10}{.5}$

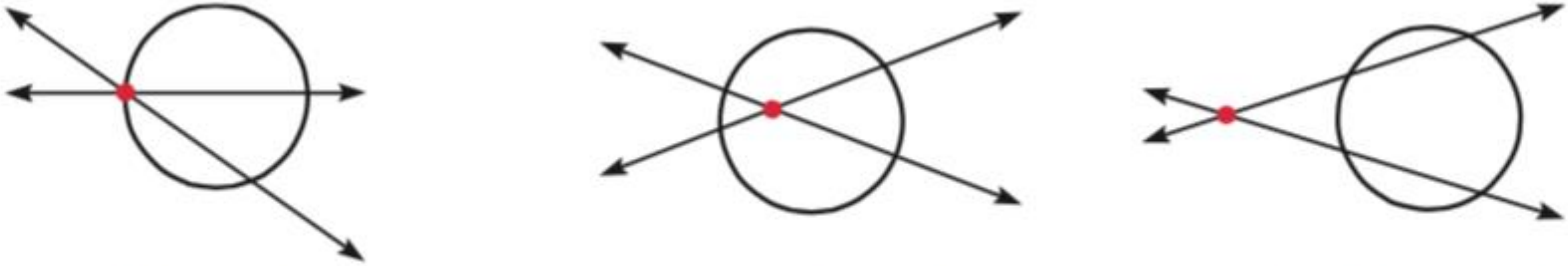
$x = 20$

$\Rightarrow \angle = 5x \rightarrow 5(20) = 100^\circ$



GOAL 2: Lines Intersecting Inside or Outside a Circle

If two lines intersect a circle, there are three places where the lines can intersect.



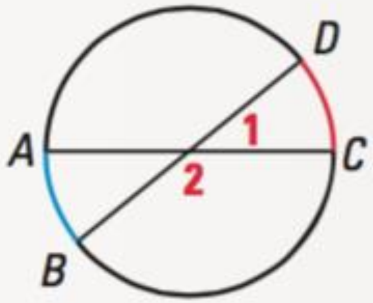
on the circle **inside the circle** **outside the circle**

You know how to find angle and arc measures when lines intersect *on* the circle. You can use Theorems 10.13 and 10.14 to find measures when the lines intersect *inside* or *outside* the circle. You will prove these theorems in Exercises 40 and 41.

THEOREMS

THEOREM 10.13

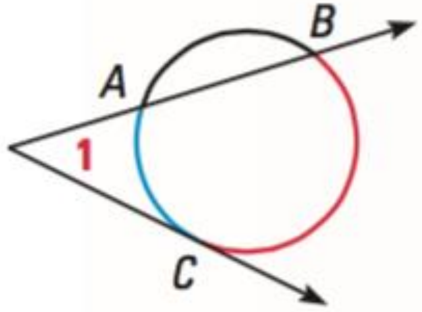
If two chords intersect in the *interior* of a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.

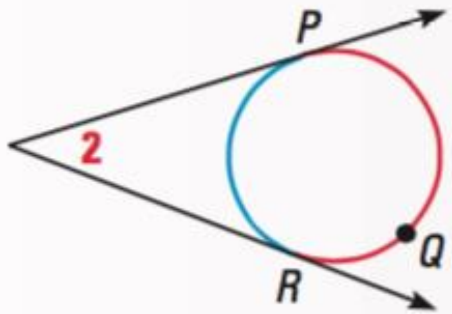


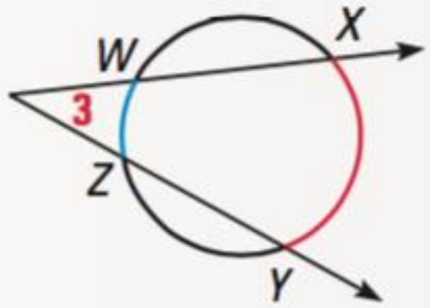
$$m\angle 1 = \frac{1}{2}(m\overparen{CD} + m\overparen{AB}),\ m\angle 2 = \frac{1}{2}(m\overparen{BC} + m\overparen{AD})$$

THEOREM 10.14

If a tangent and a secant, two tangents, or two secants intersect in the *exterior* of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.







$$m\angle 1 = \frac{1}{2}(m\overparen{BC} - m\overparen{AC})$$

$$m\angle 2 = \frac{1}{2}(m\overparen{PQR} - m\overparen{PR})$$

$$m\angle 3 = \frac{1}{2}(m\overparen{XY} - m\overparen{WZ})$$

Example 3: Finding the Measure of an Angle Formed by Two Chords

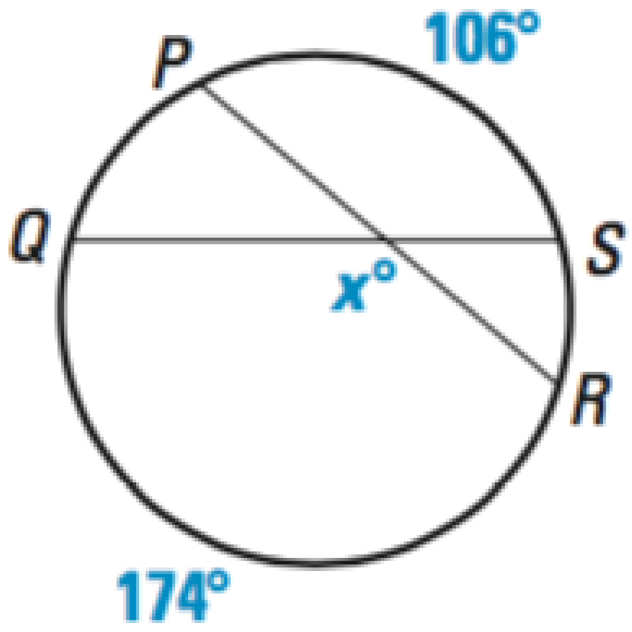
Find the value of x.

$\angle = \frac{1}{2}(\frown + \frown)$

$\angle = \frac{1}{2}(174 + 106)$

$\angle = \frac{1}{2}(280)$

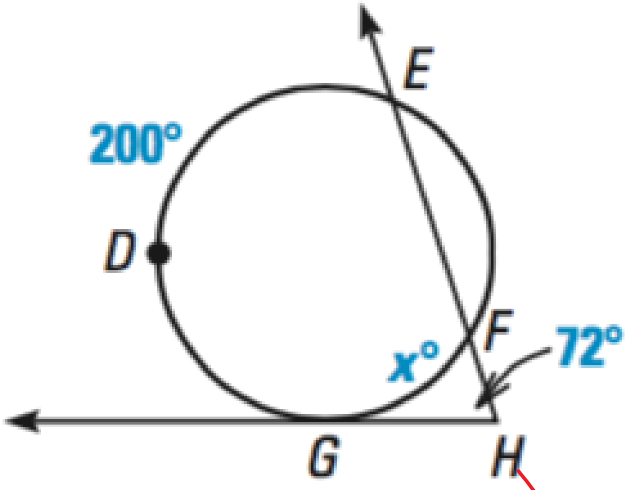
$\angle = 140^\circ$



Example 4: Using Theorem 10.14

Find the value of x.

a.



$\angle = \frac{1}{2}(\frown - \frown)$

$72 = \frac{1}{2}(200 - x)$

$72 = 100 - .5x$

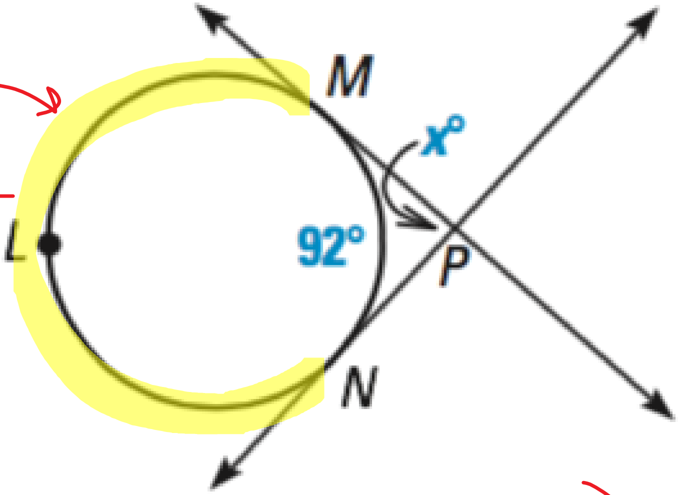
$-100 \quad -\cancel{100}$

$\frac{-28}{-.5} = \frac{-.5x}{-.5} \Rightarrow 56 = x$

b.

360−92

268°



$\angle = \frac{1}{2}(\frown - \frown)$

$\angle = \frac{1}{2}(268 - 92)$

$\angle = \frac{1}{2}(176)$

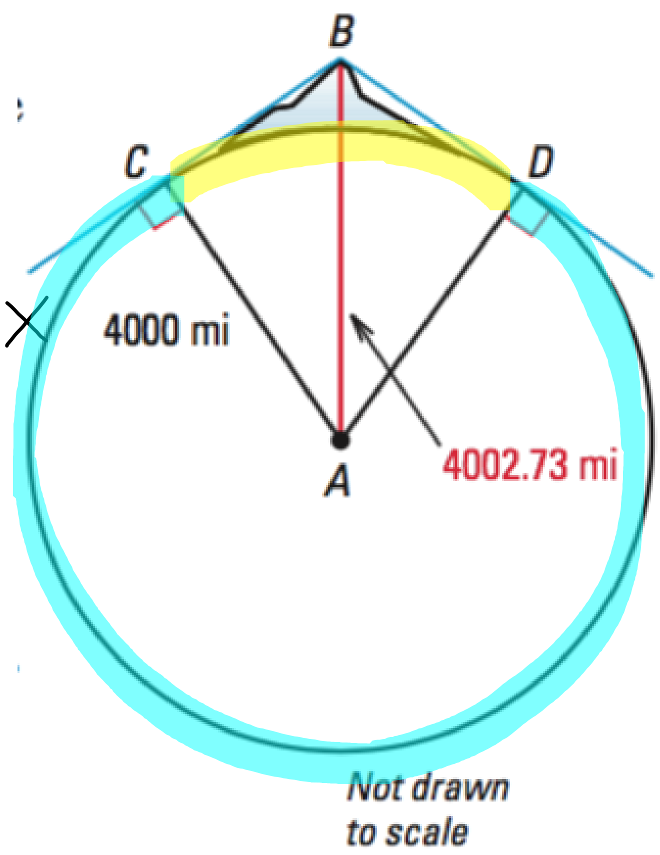
$\angle = 88°$

Example 5: Describing the View from Mount Rainier

REAL LIFE VIEWS You are on top of Mount Rainier on a clear day. You are about 2.73 miles above sea level. Find the measure of the arc $\overset{\frown}{CD}$ that represents the part of Earth that you can see.

$= x$

$= 360 - x$



$\angle CBA \rightarrow \sin^{-1}\left(\frac{4000}{4002.73}\right)$

$= 87.9$

$\Rightarrow \angle CBD = 175.8°$

$\angle = \frac{1}{2}(\frown - \frown)$

$175.8 = \frac{1}{2}(360 - x - x)$

$175.8 = \frac{1}{2}(360 - 2x)$

$175.8 = \cancel{180} - x$

$-180 \quad \cancel{-180}$

$\frac{-4.2}{-1} = \frac{\cancel{-}x}{\cancel{-1}} \Rightarrow 4.2 = x$

EXIT SLIP